

Numerical Implementation of Generalized Unitarity

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Abstract

We present a numerical implementation of generalized unitarity. We will show that we are able to obtain the box coefficients of any 1 loop gluonic amplitudes for an arbitrary helicity configuration and for any number of external particles.

1 Introduction

One loop amplitudes is an important and very complicated part of a scattering cross section calculation in high energy physics. Even for a small number of external particles the number of Feynman diagrams grows really fast, when one adds just one more leg. During the last years, there has been a lot of progress in understanding the complex structure of 1 loop amplitudes and new methods have helped us to work around Feynman diagrams in getting to the result. These methods depend on the complex plane structure of an amplitude when we continue all momenta to become complex.

The particular method that we focus, *generalized unitarity*, is a clever method of obtaining the coefficients of the master integrals, that constitute the diverging part of an amplitude. Based on cutting the propagators in the loop and obtaining a particular coefficient as product of tree amplitudes, the method lends itself naturally for an algorithmic implementation. We have developed just such an implementation, which (as a first step) computes the box coefficients of any 1 loop amplitude for any helicity configuration of the external particle. Since, implicitly, the method relies on the calculation of tree amplitudes with various helicity configurations, we have also numerically implemented the BCFW recursion equation.

2 The BCFW recursion equations

In this section we briefly present the BCFW recursion and its numerical implementation. But first let us agree on some definitions and convention that we use throughout this work.

2.1 Spinor definitions and conventions

It is a well established fact how a massless four-vector can be written as matrix using the Pauli matrices σ^μ :

$$p_\mu \longrightarrow p_{A\dot{B}} = p_\mu \sigma^\mu_{A\dot{B}} \quad (1)$$

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Being a massless vector, the *matrix* $p_{A\dot{B}}$ has a determinant zero $\det(p) = 0$. It is known from linear algebra that a matrix with zero determinant can be written as a product of two columns which we call spinors:

$$p_{A\dot{B}} = \lambda_A \tilde{\lambda}_{\dot{B}} \quad (2)$$

We write these in a convenient bra-ket form as follows:

$$\lambda_A(p_i) \rightarrow |i\rangle \quad , \quad \tilde{\lambda}_{\dot{B}}(p_i) \rightarrow \langle i| \quad (3)$$

For definite helicities we can write these spinors as solutions of the Weyl equation:

$$|i\pm\rangle = u_{\pm}(p_i) \quad \langle i\pm| = \bar{u}_{\pm}(p_i) \quad (4)$$

With these definitions we can define inner products with spinors. There are two versions the ‘angle’ product and the ‘bracket’. For real momenta these are conjugate of each other. For complex momenta this may no longer hold.

$$\langle ij\rangle = \langle i-|j+\rangle = \bar{u}_-(p_i)u_+(p_j) \quad [ij] = \langle i+|j-\rangle = \bar{u}_+(p_i)u_-(p_j) \quad (5)$$

From the definitions of the spinors we can see that products like $\langle i+|j+\rangle$ or $\langle i-|j-\rangle$ vanish. We can use these definitions to construct more complicated objects, with momenta sandwiched between spinors:

$$\langle i+|k|j+\rangle = \langle i+|\not{k}|j+\rangle = [ik]\langle kj\rangle \quad \langle i-|k|j-\rangle = \langle i-|\not{k}|j-\rangle = \langle ik\rangle[kj] \quad (6)$$

with a generalization for n momenta,

$$\langle i+|k_1 k_2 \cdots k_n|j+\rangle = \langle i+|\not{k}_1 \cdots \not{k}_n|j+\rangle = [ik_1]\langle k_1 k_2\rangle[k_2 k_3] \cdots \langle k_{n-1} k_n\rangle \quad (7)$$

We will also use the identities:

$$\langle i+|k_1 + k_2 + \cdots + k_m + \cdots + k_n|j+\rangle = \sum_{m=1}^n [im]\langle mj\rangle \quad (8)$$

$$\langle i-|k_1 + k_2 + \cdots + k_m + \cdots + k_n|j-\rangle = \sum_{m=1}^n \langle im\rangle[mj] \quad (9)$$

Finally, as you may have already noticed, we will use the index of the momentum as the symbol for the spinor, when there is no confusion.

2.2 Analytic part

For a LO amplitude, and concentrating on gluons only for simplicity, the color information can be factorized from the kinematical part as follows:

$$\mathcal{A}_n^{tree}(\{k_i, \lambda_i\}) = g^{n-2} \sum_{\sigma \in S_n/Z_n} Tr(T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} \cdots T^{a_{\sigma(n)}}) A_n^{tree}(\sigma(1^{\lambda_1}), \dots, \sigma(n^{\lambda_n})) \quad (10)$$

where T^{a_i} are the $SU(N)$ matrices in the adjoint representation. The sum is over all *non-cyclic* permutations of the external legs. The part of the factorization $A_n^{tree}(\sigma(1^{\lambda_1}), \dots, \sigma(n^{\lambda_n}))$ now depends only on the external momenta, and the polarization vectors and is in a *color ordered* form. This has the nice property that all poles that appear in it are made of adjacent momenta, s_{12}, s_{23} etc. An almost identical result holds for quarks as well.

Some years ago, Britto, Cachazo and Feng [1], showed that the kinematical part of the amplitude, the so called *partial amplitude*, has a nice factorization property that satisfies the recursion equation (thereafter known as the BCFW recursion):

$$A_n^{tree}(1, 2, \dots, n) = \sum_{h=\pm} \sum_{k=2}^{n-2} A_{k+1}^{tree}(\hat{1}, 2, \dots, -\hat{K}_{1,k}^{-h}) \frac{1}{K_{1,k}^2} A_{n-k+1}^{tree}(\hat{K}_{1,k}^h, \dots, n-1, \hat{n}) \quad (11)$$

The color ordered amplitude is split into two terms, consisting of lower point ordered amplitudes, joined by a propagator $K_{1,k} = p_1 + p_2 + \dots + p_k$. With an appropriate **complex** shift of the momenta the propagator can be put on shell. The simplest way to do that is by shifting the first and the last leg (denoted by a hat in the recursion), by a fixed amount with a complex constant z :

$$\begin{aligned} 1 &\rightarrow \hat{1} = 1 + z_k n & \tilde{1} &\rightarrow \tilde{1} \\ n &\rightarrow n & \tilde{n} &\rightarrow \hat{n} = \tilde{n} - z_k \tilde{1} \end{aligned} \quad (12)$$

The z is chosen in such a way so that the *shifted* propagator is massless:

$$z_k = -\frac{K_{1,k}^2}{\langle n - |K_{1,k}|1 \rangle} \quad (13)$$

$$\hat{K}_{1,k}^2 = (K_{1,k} + z_k n \tilde{1})^2 = K_{1,k}^2 + z_k \langle n - |K_{1,k}|1 \rangle = 0 \quad (14)$$

The sums are over all possible distributions of the external momenta into the two groups, keeping the special, shifted momenta in distinct groups, and over the helicities of the common propagator. The recursion was proven by Britto et al [2] using the analytic properties of the amplitude and by Lazopoulos et al [3] using standard Feynman diagrams analysis.

The advantage of the BCFW recursion is that only a few terms are needed to compute a partial amplitude, compared to hundreds of Feynman diagrams using standard field theory techniques. This signals major cancellations between graphs that are automatically taken care of using the recursion. Thus, we arrive at relatively compact expressions for the amplitudes.

2.3 Numerical implementation

We have implemented the BCFW recursion, in a numerical `FORTTRAN90` code, that computes the partial amplitudes for QCD, including both quarks and gluons, for an arbitrary number of external legs. The only inputs in the code is the number of legs n , the QCD process, and the helicity configuration. We have compared the results, both for MHV and NMHV helicity configurations with known results from the literature and with `HELAC` [4]. We have also produced some new numerical results. Some results are shown in the following tables for 8 and

9 point amplitudes with and without quarks. We have used the following momenta values for the 8-point amplitude:

$$\begin{aligned}
p_1 &= (50.0, 0.0, 0.0, 50.0) \\
p_2 &= (50.0, 0.0, 0.0, -50.0) \\
p_3 &= (-11.1703767834197, -8.03205150805747, 6.45958520425105, -4.30548778241159) \\
p_4 &= (-29.3374489041782, -0.341091815726772, 5.38045260205076, 28.8378274905652) \\
p_5 &= (-8.93691541288138, -7.79003576692047, 2.54732281477208, -3.56299681791019) \\
p_6 &= (-4.49061013802485, -2.19138756179328, 0.744707846345424, 3.84822169181309) \\
p_7 &= (-9.04125786194674, 4.56116168271723, 7.76548604819915, 0.798357229284189) \\
p_8 &= (-37.0233908995491, 13.7934049697808, -22.8975545156185, -25.6159218113407)
\end{aligned}$$

Helicity Configuration	Partial Amplitude
$(1^-, 2^-, 3^-, 4^-, 5^+, 6^+, 7^+, 8^+)$	$1.143995504515313 \cdot 10^{-4} + 2.452584716117081 \cdot 10^{-5}i$
$(1^-, 2^+, 3^+, 4^-, 5^-, 6^+, 7^+, 8^-)$	$-3.892700175442802 \cdot 10^{-8} - 7.138278322124297 \cdot 10^{-9}i$
$(1^-, 2^-, 3^-, 4^+, 5^+, 6^+, 7^+, 8^+)$	$2.973212107227238 \cdot 10^{-8} + 1.495525686984027 \cdot 10^{-7}i$
$(1^-, 2^-, 3^+, 4^+, 5^-, 6^-, 7^+, 8^+)$	$-8.620198212038414 \cdot 10^{-8} + 8.089172271501075 \cdot 10^{-6}i$
$(1^+, 2^+, 3^+, 4^-, 5^+, 6^-, 7^+, 8^-)$	$1.899386258601871 \cdot 10^{-6} - 9.145607147903538 \cdot 10^{-6}i$
$(1_q^-, 2_{\bar{q}}^+, 3^+, 4^+, 5^+, 6^-, 7^-, 8^-)$	$-1.812332579250756 \cdot 10^{-7} + 1.534899358766671 \cdot 10^{-7}i$
$(1_{\bar{q}}^-, 2_q^+, 3^-, 4^+, 5^-, 6^+, 7^-, 8^+)$	$-5.724656620620708 \cdot 10^{-9} - 8.875798503634412 \cdot 10^{-10}i$

Table 1. Results for 8-point partial amplitudes. Quarks and antiquarks are denoted as subscripts q and \bar{q} in the relevant legs

The following momenta configuration was used for the 9-point amplitudes:

$$\begin{aligned}
p_1 &= (50.0, 0.0, 0.0, 50.0) \\
p_2 &= (50.0, 0.0, 0.0, -50.0) \\
p_3 &= (-8.62171654444322, -6.90534943082250, 4.59215344257906, -2.35844698282037) \\
p_4 &= (-27.8413520112470, -0.643296108254438, 0.840299169363624, 27.8212319863308) \\
p_5 &= (-7.13369828021707, -6.67842455614503, 1.44431114936003, -2.04994192719828) \\
p_6 &= (-4.22769845708132, -1.90930333047348, 7.776353148874293 \cdot 10^{-2}, 3.77119979176668) \\
p_7 &= (-7.10934176699029, 3.77076042627538, 5.74622866522395, 1.81795552598701) \\
p_8 &= (-30.9459371413082, 11.4021700192282, -22.0566594617972, -18.4701195933519) \\
p_9 &= (-14.1202557987129, 0.963442980191897, 9.35590350378182, -10.5318788007139)
\end{aligned}$$

Helicity Configuration	Partial Amplitude
$(1^-, 2^-, 3^-, 4^-, 5^+, 6^+, 7^+, 8^+, 9^+)$	$-1.195041826437500 \cdot 10^{-6} - 4.267905334966595 \cdot 10^{-6}i$
$(1^-, 2^+, 3^+, 4^-, 5^-, 6^+, 7^+, 8^-, 9^-)$	$9.960596678335883 \cdot 10^{-9} + 2.632108576287432 \cdot 10^{-10}i$
$(1^-, 2^-, 3^-, 4^+, 5^+, 6^+, 7^+, 8^+, 9^+)$	$-2.628610959245734 \cdot 10^{-10} - 3.014249899217379 \cdot 10^{-9}i$
$(1^-, 2^-, 3^+, 4^+, 5^-, 6^-, 7^+, 8^+, 9^-)$	$-8.470520978313771 \cdot 10^{-7} - 1.528306470088502 \cdot 10^{-6}i$
$(1^+, 2^+, 3^+, 4^-, 5^+, 6^-, 7^+, 8^-, 9^+)$	$-1.935502160225642 \cdot 10^{-7} + 7.283711249910175 \cdot 10^{-8}i$
$(1_{\bar{q}}, 2_{\bar{q}}^+, 3^+, 4^+, 5^+, 6^-, 7^-, 8^-, 9^-)$	$-4.751617821378328 \cdot 10^{-8} - 5.102361545628036 \cdot 10^{-8}i$
$(1_{\bar{q}}, 2_{\bar{q}}^+, 3^-, 4^+, 5^-, 6^+, 7^-, 8^+, 9^-)$	$-4.359890532350093 \cdot 10^{-9} + 1.991698153552900 \cdot 10^{-9}i$

Table 2. Results for 9-point partial amplitudes. Notations are the same as before

3 The next step: NLO

The color factorization structure for the LO amplitudes, persists for NLO amplitudes only slightly more involved:

$$\begin{aligned}
\mathcal{A}_n^{1-loop}(\{k_i, \lambda_i\}) &= g^n \sum_{\sigma \in S_n/Z_n} G_{n;1}(\sigma) A_{n;1}(\sigma(1^{\lambda_1}), \dots, \sigma(n^{\lambda_n})) \\
&+ g^n \sum_{c=2}^{n/2+1} \sum_{\sigma \in S_n/S_{n;c}} G_{n;c}(\sigma) A_{n;c}(\sigma(1^{\lambda_1}), \dots, \sigma(n^{\lambda_n}))
\end{aligned} \tag{15}$$

where $G_{n;1}(\sigma)$ is the leading color structure:

$$G_{n;1}(\sigma) = N_c \text{Tr}(T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} \dots T^{a_{\sigma(n)}}) \tag{16}$$

and $G_{n;c}(\sigma)$ is the subleading color structure:

$$G_{n;c}(\sigma) = \text{Tr}(T^{a_{\sigma(1)}} \dots T^{a_{\sigma(c-1)}}) \text{Tr}(T^{a_{\sigma(c)}} \dots T^{a_{\sigma(n)}}) \tag{17}$$

The colored ordered amplitudes of the subleading terms, $A_{n;c}(\sigma)$, can be written as linear combinations of permumations of the leading color amplitudes $A_{n;1}(\sigma)$, so the latter are called *primitive amplitudes*. For the rest of this paper we focus on these primitive amplitudes.

By restricting ourselves to cyclical ordered primitive amplitudes we reduce the labour of computing hundrends or thousands of Feynman diagrams to a reduced set of diagrams. Now each of those diagrams is an integral over the loop momentum. The integrands of those integrals are made of tensors or vectors of the external momenta and the loop momenta. It would an enormous simplification if we could further reduce these to a basic set of integrals, so that every diagram could be written as a combination in this basis. It turns out that such a reduction is possible, either using standard Passarino-Veltman techniques [5], or the more recent Ossola-Papadopoulos-Pittau [6] reduction method. Thus all loop integrals can be brought down to a basis consisting of up to 4 propagators:

$$\mathcal{I} = \{I_2, I_3^{1m}, I_3^{2m}, I_3^{3m}, I_4^{1m}, I_4^{2m}, I_4^{2m}, I_4^{2m}, I_4^{3m}, I_4^{4m}\} \tag{18}$$

As a result any 1 loop amplitude can be written as a linear combination in this basis, with *algebraic* coefficients.

$$A_n^{1-loop} \sim \sum_j a_j \mathcal{I}_j + \mathcal{R}_n \quad (19)$$

where \mathcal{R}_n are the rational terms. More explicitly we can write:

$$A_n^{1-loop} \sim \sum_i b_i B(K_i^2) + \sum_{ij} c_{ij} C(K_i^2, K_j^2) + \sum_{ijk} d_{ijk} D(K_i^2, K_j^2, K_k^2) + \mathcal{R}_n \quad (20)$$

where B, C and D are the bubble, triangle and box integrals respectively. Analytic expressions and singularity structure for these integrals can be found for example in [7]

4 Generalized Unitarity

Unitarity of the scattering matrix in field theory implies the conservation of probability. On the amplitude level unitarity says that cutting a loop gives the discontinuity in the scattering amplitude. By cutting we mean putting a propagator on shell, which amounts in the replacement:

$$\frac{i}{p^2 + i\epsilon} \rightarrow 2\pi\delta^{(+)}(p^2) \quad (21)$$

At the diagrammatic level this is the well known Cutkosky rule. In general, at the amplitude level, cutting an amplitude in a given channel isolates those integrals that have a discontinuity across that channel. In other words cutting gives a linear combination of master integrals with algebraic coefficients. If we wish to isolate one single integral we must go a bit further and insist *more* propagators go on shell. This goes under the name of *generalized unitarity* [8]. For the box integrals in particular this is quite easy to do. Cutting four propagators, isolates a single box integral. The cut breaks the loop integral in four tree level amplitudes (one at each corner of the box) and the coefficient of the box integral is simply the product of those tree amplitudes:

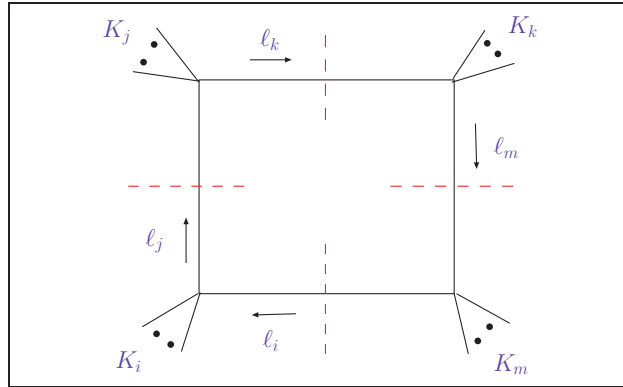


Fig.1 Cutting 4 propagators, gives the Box coefficient, as a product of the tree level

$$d_{ijk} = \frac{1}{2} \sum_{a=1}^2 \sum_{h=\pm} A_1^{tree}(\ell_i^{h_i}, \{i\}, -\ell_j^{-h_j}) A_2^{tree}(\ell_j^{h_j}, \{j\}, -\ell_k^{-h_k}) \\ \times A_3^{tree}(\ell_k^{h_k}, \{k\}, -\ell_m^{-h_m}) A_4^{tree}(\ell_m^{h_m}, \{m\}, -\ell_i^{-h_i}) \quad (22)$$

where $\{i\}, \{j\}$ etc., denotes the collection of momenta in that vertex of the box. One also averages over the two solutions of the quadratic equation one obtains by putting four propagators on shell. Finally there is a sum over the helicities of the propagators.

Simple example $A(1^+, 2^+, 3^-, 4^-)$ We have to satisfy the on-shell conditions: $l_1^2 = 0, l_2^2 = 0, l_3^2 = 0, l_4^2 = 0$. Choosing l_2 to be : $l_2^\mu = \xi \langle 1 - |\gamma^\mu| 2 - \rangle$, where ξ is a normalization, satisfies the first three. The last will determine ξ . The product of the trees in the corners of the box is:

$$\frac{1}{2} \frac{[1l_2]^3}{[l_2l_1][l_11]} \frac{\langle l_44 \rangle^3}{\langle 4l_1 \rangle \langle l_1l_4 \rangle} \frac{\langle l_3l_2 \rangle^3}{\langle l_22 \rangle \langle 2l_3 \rangle} \frac{[l_4l_3]^3}{[l_33][3l_4]} \quad (23)$$

We can combine the numerator into one factor:

$$[1l_2]\langle l_2l_3 \rangle [l_3l_4]\langle l_44 \rangle = \langle 1 + |l_2l_3l_4|4+ \rangle \quad (24)$$

Using the identities $l_3l_4 = l_3(l_3 + \not{3}) = l_3\not{3}$ and $l_2l_3l_4 = l_2l_3\not{3} = l_2(l_2 + \not{2})\not{3} = l_2\not{2}\not{3}$ this can be simplified:

$$\langle 1 + |l_2l_3l_4|4+ \rangle = \langle 1 + |l_223|4+ \rangle = \langle 1 + |l_2|2+ \rangle [23]\langle 34 \rangle = s_{12}[23]\langle 34 \rangle \quad (25)$$

Combining appropriately factors in the denominator, we can simplify the expression:

$$[3l_4]\langle l_4l_1 \rangle [l_1l_2]\langle l_22 \rangle = \langle 3 + |l_4l_1l_2|2+ \rangle = [34]\langle 41 \rangle \langle 1 + |l_2|2+ \rangle = [34]\langle 41 \rangle s_{12} \quad (26)$$

$$\langle 4l_1 \rangle [l_11] = \langle 4 - |l_1|1- \rangle = \langle 41 \rangle [21] \quad (27)$$

$$\langle 2l_3 \rangle [l_33] = \langle 2 - |l_3|3- \rangle = \langle 21 \rangle [23] \quad (28)$$

Putting together numerator and denominator and using conservation of momenta we finally get:

$$d_{1:2:3:4} = \frac{1}{2} s_{12}s_{23} A_4^{tree}(1^+, 2^+, 3^-, 4^-) \quad (29)$$

5 Numerical Implementation

We have implemented generalized unitarity in a numerical **FORTRAN 95** code. For the time being it has been implemented for box coefficients only and for purely gluonic amplitudes (no quarks in the loop). The evaluation of the box coefficient goes through the following steps:

- **Lists all possible groupings for the external momenta:** The external momenta are grouped in 4 groups (the corners of the box), in all possible, distinct ways. Sums of these momenta make up the channel that the cut is computed.

- **Solutions of the onshell loop momenta condition:** The system of equations: $\ell_1^2 = 0, \ell_2^2 = 0, \ell_3^2 = 0, \ell_4^2 = 0$ for the cut loop momenta is solved and the two solutions, that we have to average over are evaluated
- **The tree amplitudes are computed:** For each corner of the box we compute the tree amplitude defined by the external momenta in that corner, the solution for the loop momentum that we just obtained and the particular helicity configuration of the grouping of the external momenta. The computation of the tree amplitudes is done automatically using the numerical code for the BCFW recursion that we discussed in a previous section.
- Finally the coefficient for that particular cut is computed, using Eq. (22).

We have produced some results for two 1-loop processes, namely $gg \rightarrow 8g$ and $gg \rightarrow 9g$ for various choices of helicity configurations and for various channels (particular coefficients of a master integral). We have used the same momenta configuration as in the BCFW recursion section. We have compared our results with the ones in the literature and we have also produced some new numerical results in the case of 1-loop 9 gluon amplitudes. The results are shown in the following tables.

	- + - + - + - +	- - - - + + + +
(1 : 2 : 3 : 45678)	$-0.452833202541873 + 1.83433531772824i$	$564.146943978796 + 341.450920347787i$
(1 : 23 : 4 : 5678)	$-1.51120109392466410^{-5} - 2.69252629629608410^{-5}i$	$85.7803228327941 - 4.79606435167831i$
(1 : 2 : 345 : 678)	$-0.596872851993693 - 1.13690332085509i$	$366.912336610440 + 98.0271230853793i$
(1 : 2 : 34 : 5678)	$0.343744213463153 - 0.116177716850307i$	$0.0 + 0.0i$
(12 : 34 : 56 : 78)	$-6.08936506243673310^{-3} - 0.185567043177724i$	$0.0 + 0.0i$

	- + - + - + + +	- + - + + + - +
(1 : 2 : 3 : 45678)	$-1.810002621799826 \cdot 10^{-2} - 7.007553695597921 \cdot 10^{-5}i$	$1.282527393614916 \cdot 10^{-3} - 1.544463085586650 \cdot 10^{-2}i$
(1 : 23 : 4 : 5678)	$-9.061239878511741 \cdot 10^{-7} - 6.182682488354774 \cdot 10^{-7}i$	$-3.593566469301429 \cdot 10^{-6} - 2.807114551959228 \cdot 10^{-6}i$
(1 : 2 : 345 : 678)	$-6.198557976336275 \cdot 10^{-7} - 3.145711006839414 \cdot 10^{-7}i$	$1.201810688581596 \cdot 10^{-3} + 1.747770080527506 \cdot 10^{-4}i$
(1 : 2 : 34 : 5678)	$2.057234610074154 \cdot 10^{-3} + 5.080380495579885 \cdot 10^{-5}i$	$0.0 + 0.0i$
(12 : 34 : 56 : 78)	$0.0 + 0.0i$	$0.0 + 0.0i$

Table 3. Results for 1-loop, 8-point primitive amplitudes.

The code provides **all** the coefficients for a particular process at once. Typical times for the two processes shown in the tables are of the order of 1/10th of a second on a 64-bit Intel Core Duo at 2GHz.

	- + - + - + - + -	- - - - + + + +
(1 : 2 : 3 : 456789)	$-0.291597431159727 + 0.263428066914340i$	$-9.42379749599236 + 8.63580103044842i$
(1 : 23 : 4 : 56789)	$6.917959935625083 \cdot 10^{-7} - 2.862781163146907 \cdot 10^{-7}i$	$-4.424679500634447 \cdot 10^{-2} - 1.06642829663490i$
(1 : 2 : 345 : 6789)	$1.83770214755652 + 1.25751379616759i$	$-3.16966894074953 - 10.5119953247915i$
(1 : 2 : 34 : 56789)	$-0.801669928958509 - 0.704349268560792i$	$0.0 + 0.0i$
(12 : 34 : 56 : 789)	$4.147713723224443 \cdot 10^{-2} + 3.388946250063977 \cdot 10^{-2}i$	$0.0 + 0.0i$

	- + - + - + + + -	- + - + + + - + -
(1 : 2 : 3 : 456789)	$-2.382052305295742 \cdot 10^{-3} - 1.320540695786225 \cdot 10^{-2}i$	$7.517076253761092 \cdot 10^{-3} - 2.651249643255122 \cdot 10^{-3}i$
(1 : 23 : 4 : 56789)	$2.330322361904153 \cdot 10^{-8} - 2.184457500759925 \cdot 10^{-8}i$	$7.985298353983870 \cdot 10^{-8} - 1.068015483724008 \cdot 10^{-7}i$
(1 : 2 : 345 : 6789)	$1.731650767307649 \cdot 10^{-2} - 1.381732810170124 \cdot 10^{-2}i$	$1.377043202412121 \cdot 10^{-4} + 1.023698942546792 \cdot 10^{-4}i$
(1 : 2 : 34 : 56789)	$-9.470193967875743 \cdot 10^{-3} + 8.652362025481619 \cdot 10^{-3}i$	$-5.650383858159608 \cdot 10^{-4} - 1.479340069213934 \cdot 10^{-4}i$
(12 : 34 : 56 : 789)	$-8.488224296845019 \cdot 10^{-5} - 1.761227642104737 \cdot 10^{-4}i$	$3.287516684614978 \cdot 10^{-8} + 1.134952815474561 \cdot 10^{-7}i$

Table 4. Results for 1-loop, 9-point primitive amplitudes

Triangle coefficients can be obtained in much the same way. Cutting 3 propagators does not immediately isolate a single integral but it gives a triangle integral plus a sum of box integrals [9]. This is because the particular box integrals share the same cuts with the triangles. The boxes have to be subtracted in a suitable way. This can naturally be implemented in our algorithm since box coefficients are computed already. For bubble integrals, cutting 2 propagators and subtracting the triangle and box contributions will isolate a single coefficient. Work is in progress to implement just this procedure.

6 Summary and Outlook

We presented numerical implementations for the BCFW recursion equations and 1-loop coefficients of box integrals using generalized unitarity. In the case of the BCFW recursion equations we were able to provide, in a fast algorithmic way, multi-particle amplitudes both old and new, namely in the case of 9 parton Leading Order amplitude. For the implementation of generalized unitarity we are able to compute all box coefficients at once for an arbitrary number of particles and helicity configuration. Some new coefficients for the 1 loop 9 gluon amplitude were presented. Work is in progress to include quarks in the picture (easy step since this is already done in the BCFW recursion code) and also complete the implementation by computing triangle and bubble coefficients.

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